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The spheric aberration of the nonlinear GRIN lens using ray-transfer matrix

Xác định cầu sai của thấu kính GRIN phi tuyến bằng cách sử dụng ma trận truyền tia

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Abstract

The nonlinear GRIN lens is an optical element proposed to enhance the optical trap efficiency and flexibility of the nonlinear optical tweezers. The optical quantity of the nonlinear GRIN lens has been evaluated by the paraxial approximation, so there is a suspicion of its optical quality, especially spherical aberrations.. In this paper, we have suggested that the induced index of the nonlinear medium described by the Maclaurin's formula of the Gaussian function, then using this induced index to investigate the spheric aberration of the nonlinear GRIN lens. The expressions describing the spheric aberration are derived based on the comparison of the output radial distance of the ideal GRIN lens and its reality. The numerically calculated results are applied for the high nonlinear organic dye layer. The capability to correct the spheric aberration is discussed.

Keywords: Nonlinear optics; optic devices; aspheric surface; optic aberration.

Tóm tắt

Thấu kính GRIN phi tuyến tính là một thành phần quang học được đề xuất để nâng cao hiệu quả bẫy quang học và tính linh hoạt của kìm quang học phi tuyến. Tính chất quang học của thấu kính GRIN phi tuyến đã được xem xét bằng phép tính gần đúng trục, do đó, cần xem xét đánh giá về chất lượng quang học của nó, đặc biệt là cầu sai. Trong bài báo này, chúng tôi đã đề xuất rằng chiết suất của môi trường phi tuyến được mô tả bởi công thức Maclaurin của hàm Gaussian, sau đó khảo sát sự cầu sai của thấu kính GRIN phi tuyến. Các biểu thức mô tả quang cầu sai được suy ra dựa trên sự so sánh giữa khoảng cách bán kính đầu ra của thấu kính GRIN lý tưởng và thấu kính thực tế của nó. Các kết quả tính toán bằng số được áp dụng cho lớp màng mỏng hữu cơ có tính phi tuyến cao. Khả năng sửa cầu sai được thảo luận.

Từ khóa: Quang phi tuyến; thiết bị quang học; bề mặt phi cầu; quang sai.

1. Introduction

Under the irradiation of the laser Gaussian beam (LGB), a Kerr medium (KM) becomes the GRIN medium [1]]. Up to now, the organic dye thin layer (ODTL) owning high nonlinearity [2,3,4] has been proposed to use as a nonlinear GRIN lens (NGL) for the nonlinear optical tweezers [5, 6], nonlinear microscope [7,8], bistable device [9] and spatial modulator of the LGB [10]. However, because of the

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paraxial approximation, there is a suspicion of the optical aberration, particularly spheric aberration (SA) of the NGL. Fortunately, the SA may be to correct by the aspheric surface [11-14]. In pervious Refs. [5,6,7], the NGL is suggested as an ideal GRIN lens, then used the paraxial approximation to determine its focal length, radius of curvature. It is well known that the ideal GRIN lens do not have the SA [15], but the NGL formatted in the KM could be the nonlinear aspheric lens (NAL), based on that the induced index of the KM includes high-order compensate terms.

In this paper, we pay attention to the high-order compensate terms in the induced index of the KM irradiated by LGB. The expressions describing the SA of the NGL and NAL are derived. Next, the SA is numerically calculated for the case of Acide Blue thin layer. Last, the dependence of the SA on the design parameters are investigated and discussed.

2. Proposed model for calculation and bases of theory

The proposed model of the NGL and NAL with rays traversing through them is presented in Fig. 1. A LGB with the beam waist of radius, W_0 and intensity in axis of I_0. This beam propagates through the KM with physical thickness of d_{phys}, nonlinear coefficient of index, n_{nl} and linear index of n_0. The intensity transverse distribution of the LGB is described by the Gaussian function of the radial distance, $\rho = \sqrt{x^2 + y^2}$. From now on, all of physical quantities mentioned above are called the design parameters. Under the irradiation of laser intense light, I the refractive index of the KM will be induced and given as the following formula [16]:

$$n(I) = n_0 + n_{nl} I \tag{1}$$

Conventionally, the light intensity of the LGB in the mode TEM00 or in the region limited by Rayleigh range is given as Gaussian function:

$$I(\rho) = I_0 exp\left[-\frac{2\rho^2}{W_0^2}\right] \tag{2}$$

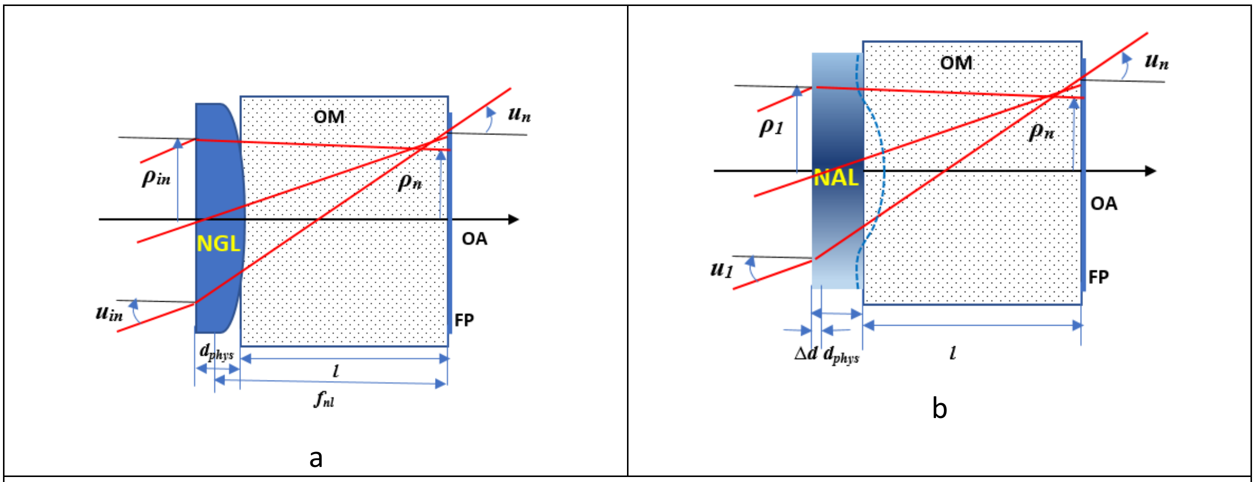

Fig.1 Rays traverse through NGL (a) and NAL (b)

Using the Maclaurin's formula for the exponential function [17], and limiting to fifth-order term, we have:

$$I(\rho) = I_0 exp\left[-\frac{2\rho^2}{W_0^2}\right] \approx I_0\left(1 - \frac{2\rho^2}{W_0^2} + \frac{2\rho^4}{W_0^4} - \frac{4\rho^6}{3W_0^6} + \frac{2\rho^8}{3W_0^8}\right) \tag{3}$$

Substituting Eq. (3) into Eq. (1) we have induced index of the KM as following:

$$n(\rho) = n_0 + n_{nl}I_0 - n_{nl}I_0\frac{2\rho^2}{W_0^2} + n_{nl}I_0\left(\frac{2\rho^4}{W_0^4} - \frac{4\rho^6}{3W_0^6} + \frac{2\rho^8}{3W_0^8}\right) \tag{4}$$

where three first terms are the major facts creating the NGL(Fig. a) [5,6,7], and the rest terms

$$d^{com} = d_{phys}n_{nl}I_0\left(-\frac{2\rho^4}{W_0^4} + \frac{4\rho^6}{3W_0^6} - \frac{2\rho^8}{3W_0^8}\right) \tag{5}$$

are the compensate playing the role of the aspheric coefficients [18, 19], which deviates the curvature surface of the NGL. With the compensate terms, the NGL will be the NAL (Fig.1b). The most important term of the optical aberration is the SA. Now, we derive the SA of the NGL and NAL.

Firstly, as shown in Ref. [5,6,7], the NGL is proposed to format in the radial region $\rho << W_0$ inside the KM. So, we apply the paraxial approximation, i.e., $I(\rho) \approx I_0\left(1 - 2\rho^2 / W_{out}^2\right)$, then the induced index of the KM is reduced to:

$$n_{sph}(\rho) = n_0 + n_{nl}I_0\left(1 - \frac{2\rho^2}{W_0^2}\right) = N_0 - \frac{N_{nl}^2}{2}\rho^2 \tag{6}$$

where $N_0 = n_0 + n_{nl}I_0$ is called the optical induced index in the OA, $N_{nl} = \frac{2}{W_0}\sqrt{n_{nl}I_0}$, which describes the gradient-index of the KM, is called the induced reducing coefficient [20, 21]. With the condition $d < \pi / 2N_{nl}$, the focal length of the NGL is given as following [7,22]:

$$f_{nl} = \frac{W_0^2}{d_{phys}n_2I_0} \tag{7}$$

Considering a ray irradiating the KM at the radial distance ρ_{in} with input angle (or ray angle) u_{in}, and then traverses through the NGL and the output medium with index, n_f. The ray meets the focal plane at the radial distance ρ_{out} with output angle u_{out} (Fig,1a). Using the ray-transfer matrix [23], the relation between the input and output radial distances and ray angles is the matrix equation given as following:

$$\begin{bmatrix} \rho_{out} \\ u_{out} \end{bmatrix} = \begin{bmatrix} 1 & l/n_f \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_{nl} & 1 \end{bmatrix}\begin{bmatrix} \rho_{in} \\ u_{in} \end{bmatrix} \tag{8}$$

From (8) we have:

$$\rho_{out} = \frac{\rho_{in}\left(f_{nl}n_f - l\right)}{f_{nl}n_f} + \frac{f_{nl}u_{in}}{n_f} \tag{9}$$

From Eq. (9), the output radial distance in the focal plane of the ray traversing through $\rho_{in} = 0$ is:

$$\rho_f^{NGL} = \frac{f_{nl}u_{in}}{n_f} \tag{10}$$

The transverse SA is defined as the difference between the output radial distances ρ_{out} and ρ_f. Using Eq.(9) and Eq.(10), the transverse SA of the NGL, $W_{NGL}^x(\rho)$ will be:

$$W_{NGL}^x(\rho) = \rho_{out} - \rho_f^{NGL} = \frac{\rho_{in}\left(f_{nl}n_f - f_{nl} + 0.5d_{phys}\right)}{f_{nl}n_f} \tag{11}$$

which depend on the ray input angle, ρ_{in} and design parameters. From Eq. (11), it is clear that if the index of output medium is of $n_f = 1$, the transverse SA will be close to zero. It is in good agreement to Ref. [15] that the SA of the ideal NGL can be neglected, only the output medium is air ($n_f \approx 1$). The SA of the NGL will be zero, only when $n_f = 1$ and $l = f_{nl}$. On the contrary, if the index of the output medium is bigger than that of air, the SA of the NGL is always different to zero. The SA is proportional to the input radial distance. Beyond the air, all of the traditional medium owns index bigger than index of the air, so the NGL always has its SA.

Secondly, we derive the relation between the input and output radial distances of the ray traversing the NAL (Fig.1b). To use the ray-transfer matrix to calculate the SA of the NAL, the KM with the physical thickness, d_{phys} will be divided into a series of the thinner layers with physical thickness $\Delta d = d_{phys}/m$, where m is an integer chosen which is suitable to suggesting accuracy. As shown in Fig.1b, the relation between output and input radial distances is given as:

$$\begin{bmatrix}\rho_n\\ u_n\end{bmatrix}=\begin{bmatrix}1 & l/n_f\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \Delta d/n(\rho_{n-1})\\ 0 & 1\end{bmatrix}\ldots\begin{bmatrix}1 & \Delta d/n(\rho_1)\\ 0 & 1\end{bmatrix}\begin{bmatrix}\rho_1\\ u_1\end{bmatrix} \qquad (12)$$

and after some arrangements, we have:

$$\rho_n = \rho_1 + u_1\Delta d\sum_{i=1}^{m}\frac{1}{n(\rho_i)}+\frac{u_1 l}{n_f} \qquad (13)$$

From Eq. (13), the output radial distance of the ray at $\rho_1 = 0$ will be:

$$\rho_f^{NAL} = u_1\Delta d\sum_{i=1}^{n-1}\left.\frac{1}{n(\rho_i)}\right|_{n(\rho_1=0)}+\frac{u_1 l}{n_f} \qquad (14)$$

where $n(\rho_1)=n(0)$.

From Eq.(13) and Eq.(14) we have the SA of the NAL, $W_{NAL}^{x}(\rho)$ as following:

$$W_{NAL}^{x}(\rho)=\rho_n-\rho_f^{NAL}=\rho_1+u_1\Delta d\sum_{i=1}^{n-1}\left.\frac{1}{n(\rho_i)}\right|_{n(\rho_1\neq 0)}-u_1\Delta d\sum_{i=1}^{n-1}\left.\frac{1}{n(\rho_i)}\right|_{n(\rho_1=0)} \qquad (15)$$

There is an important point that the SA of NGL (see Eq, (11)) depends mainly on the index of the output medium, meanwhile, that of the NAL does not (see Eq. (14)). The SA of the NAL depends on the design parameters. Using Eq. (11) and Eq. (15), the SA will be numerically calculated and discussed the difference between NGL and NAL.

3. Results and discussion

For numerically calculation, we use design parameters as: the thin layer of Acide Blue with physical thickness of $d_{phys}=(0.04\div 0.1)cm$, $n_{nl}=1.10^{-6}cm^2/W$ and $n_0=1.45$[4]; the LGB with $\lambda=1.06\mu m$, $I_0=(1.10^3\div 1.10^6)W/cm^2$, and $W_0=2cm$.

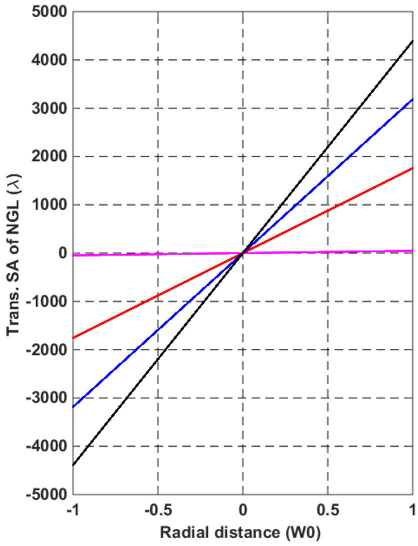

Fig.2 SA of NGL for the case of $d_{phys}=0.1cm$, $n_{nl}=1.10^{-6}cm^2/W$, $I_0=1.10^6 W/cm^2$ and different index of the output medium: $n_f=1.0$ (magenta), $n_f=1.1$ (red), $n_f=1.2$ (blue), $n_f=1.3$ (black).

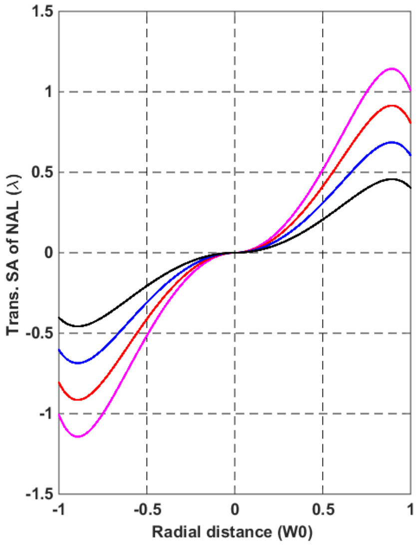

Fig.3 SA of NAL for the case of of $u_{in}=2^o$ $d_{phys}=0.1cm$, $n_{nl}=1.10^{-6}cm^2/W$, $I_0=1.10^6 W/cm^2$ and different physical thickness: $d_{phys}=0.04cm$ (black), $d_{phys}=0.06cm$ (blue), $d_{phys}=0.08cm$ (red), $d_{phys}=0.1cm$ (magenta).

Firstly, use Eq.(11) to observe the SA of the NGL. Because, the SA of the NGL mainly depends on the index of output medium, so we investigate the SA for the case of $d_{phys} = 0.1cm$, $n_{nl} = 1.10^{-6} cm^2 / W$, $I_0 = 1.10^6 W / cm^2$ and different index of the output medium. The obtained results are presented in Fig.2. From Fig.2 we see that if the output medium is air, the SA of the NGL is always different to zero and similar to that obtained for the electrically thin flat lenses [24]. For the edge ray of the laser beam waist ($W_0 = 2cm$), the $W^x_{NGL}(2cm) = 50\lambda$, which is too high for the desired aspheric lens (SA <λ/2 [25, 26]). The SA of the NGL increases powerfully and is proportional to the index of the output medium. Particularly, if index of the output is of $n_f = 1.3$, the SA of the edge ray increases to $W^x_{NGL}(2cm) = 4300\lambda$, which is not never accepted to use in the optical system. Thus, we can say that the NGL should not be used as the ideal NGL, then as the NAL only. This comment will be confirmed in the next investigation.

The SA of the NAL is calculated for the case of $u_{in} = 2^o$, $d_{phys} = 0.1cm$, $n_{nl} = 1.10^{-6} cm^2 / W$, $I_0 = 1.10^6 W / cm^2$ and different physical thickness $d_{phys} = 0.04cm$, $d_{phys} = 0.06cm$, $d_{phys} = 0.08cm$, $d_{phys} = 0.1cm$ presented in Fig.3. All results in Fig.3 show that the SA increases with the increasing of the physical thickness of the KM. This phenomena can be easily explained by the shorting of the nonlinear focal length relating to the increasing of physical thickness. The maximum value of the SA is about wavelength for the edge ray. This value may be accepted for the traditional optical system, but it is still high. Continuously, keeping $d_{phys} = 0.04cm$ and tun the laser intensity decreasing, the SA changes as shown in Fig.4. From Fig.4, it is clear that the SA decreases proportionally to the laser intensity. Its decreasing rule is explained when we deceases the physical thickness or the nonlinear coefficient of index.

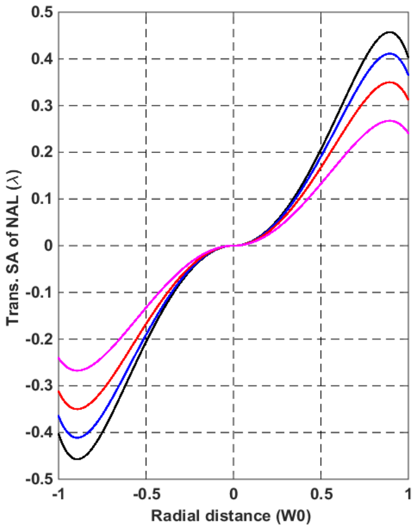

Fig.4 SA of NAL for the case of of $u_{in} = 2^o$, $d_{phys} = 0.04cm$, $n_{nl} = 1.10^{-6} cm^2 / W$, and different laser intensity: $I_0 = 1.10^6 W / cm^2$ (black), $I_0 = 0.8.10^6 W / cm^2$ (blue), $I_0 = 0.6.10^6 W / cm^2$ (red), and $I_0 = 0.4.10^6 W / cm^2$ (magenta).

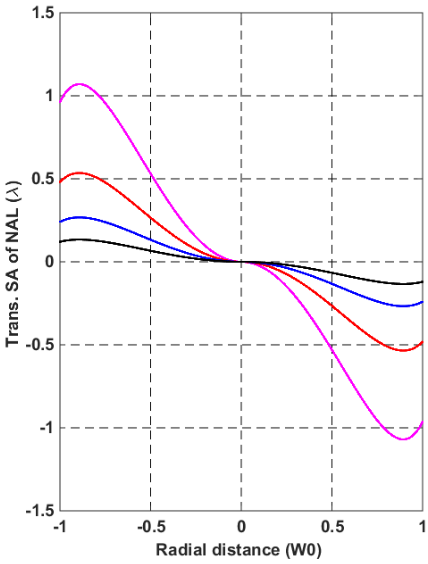

Fig.5 SA of NAL for the case of $d_{phys} = 0.04cm$, $n_{nl} = 1.10^{-6} cm^2 / W$, $I_0 = 1.10^6 W / cm^2$ and different ray-angle: $u_{in} = 1^o$ (black), $u_{in} = 2^o$ (blue), $u_{in} = 4^o$ (red), and $u_{in} = 8^o$ (magenta).

The main obtained results are that the SA are always less than the desired one, which could reach in the objective lens consisting of 6 lens with three aspheric surfaces [27], and telescope calculated with two aspheric coefficients [14]. For application in reality, we have checked the SA when the ray angle (incident angle) increases. The obtained results are in Fig.5. The Fig.5 shows us that, the SA will increase proportionally to the ray angle. However, for the case of $d_{phys} = 0.04cm$, $n_{nl} = 1.10^{-6} cm^2/W$, $I_0 = 1.10^6 W/cm^2$, the maximum value of the SA of edge ray is $W_{NGL}^{x}(2cm) \approx 1\lambda$, only when incident angle increases to $u_{in} = 8^o$. In fact, the incident is always kept less than $u_{in} = 2^o$, so, if the collection of the design parameters is chosen as following $d_{phys} = 0.04cm$, $n_{nl} = 1.10^{-6} cm^2/W$, $I_0 = 1.10^6 W/cm^2$, the SA will be neglected. The dependence of the collection of the design parameters gives us an opportunity to correct the SA of the NAL.

In summary, a nonlinear medium irradiated by the laser Gaussian beam does not seem to be the ideal NGL, but the NAL. The results obtained by using the ray-transfer matrix show clearly the SA could be corrected to the very low, less than the desired for the traditional optical system. The importance is the physical thickness of the KM and laser intensity can be reduced, then it increases the capability of the application. Moreover, the questions about the optical quality of the nonlinear optical tweezers will be cancelled.

4. Conclusion

We have used the ray-transfer matrix to introduce the SA of the NGL and NAL formatting inside the KM irradiated by the LGB. The obtained results confirm that the ideal NGL with high SA never exists, with the NAL correction able low, even negligible. The available high nonlinear organic dye and weak power laser gives us an opportunity to use the NAL for the nonlinear optical tweezers and complex optical system. The NAL seems to be SA-flexible aspheric lens which could replace the traditional solid aspheric lens in the multi-lens optical system. There is a suggestion for experimental investigation to design optical system without spheric aberration particularly, and optical aberration generally in the future.

References

[1] R.R. Krishnamurthy, R. Alkondan. (2010). Nonlinear characterization of Mercurochrome dye for potential application in optical limiting. *Opt. Appl. XL*, 187-196

[2] H.A. Badran, Q.M.A. Hassan, A.Y. Al-Ahmad, C.A. Emshary. (2011). Laser-induced optical nonlinearities in Orange G dye: polyacrylamide gel. *Canadian Journal of Physics*, *89*(12), 1219-1224.

[3] S. Jeyaram, T. Geethakrishnan. (2017). Thidr-order nonlinear optical properties of acid green 25 dye by Z-scan method. *Optics& Laser Technology*, *89*, 2570-2574.

[4] L. T. Nguyen et al. (2014). The numerical methods for analyzing the Z-scan data. *J. Nonlinear Optic. Phys. Mat. 23*, 1450020.

[5] Quy Ho Quang, Thanh Thai Doan, Tuan Doan Quoc, Thang Nguyen Manh. (2018). Enhance of optical trapping efficiency by nonlinear optical tweezers. *Opt. Commun.*, *427*, 341-347.

[6] Quy Ho Quang, Thanh Thai Doan, Tuan Doan Quoc, Thang Nguyen Manh. (2018). Nonlinear optical tweezers for longitudinal control of dielectric particles. *Opt. Commun.*, *421*, 94-98.

[7] Quy Ho Quang, Thanh Thai Doan, Tuan Doan Quoc, Viet Do Thanh, Kien Bui Xuan, Le Ly Nguyen, and Thang Nguyen Manh. (2020). Nonlinear microscope objective using thin layer of organic dye for optical tweezers. *Eur. Phys. J. D.*, 74: 52.

[8] Quy Ho Quang, Thanh Thai Doan, Tuan Doan Quoc, Le Ly Nguyen, Thang Nguyen Manh. (2020). Optimization of nonlinear optical tweezers suitable to stretch DNA molecules without broken state. *Optical and Quantum Electronics* 52:189.

[9] Quy Ho Quang, Thanh Thai Doan, Kien Bui Xuan, Thang Nguyen Manh, and Quang Ho Dinh. (2020). Optical bistability of partial reflection-coated thin film of oil red O. *Appl. Opt. 59* (19), 5664-5669.

[10] Quy Ho Quang, ThanhThai Doan, Kien Bui Xuan, Quang Ho Dinh, Thang Nguyen Manh. (2021).

Spatial modulation of Gaussian beam by dual-beam nonlinear Fabry-Perot interferometer. *Optik - International Journal for Light and Electron Optics 241*, 167248.

[11] Rafael G. González-Acuña and Héctor A. Chaparro-Romo. (2018). General formula for bi-aspheric singlet lens design free of spherical aberration. *Applied Optics Vol. 57, Issue 31*, 9341-9345.

[12] HuaQin. (2012). Aberration correction of a single aspheric lens with particle swarm algorithm. *Optics Communications 285 (*13-14*)*, 2996-3000.

[13] Antonin Miks, Petr Pokorný (2017). Spherical aberration of an optical system and its influence on depth of focus. *Applied Optics 56*(17):5099.

[14] S. Vazquez-Montiel and O. Garcıa-Lievanos. (2013). Spherical aberration correction using aspheric surfaces with an analytic-numerical method. *Revista Mexicana de Fısica* 59, 273–281.

[15] Flores-Arias, M.T.; Bao, C.; Castelo, A.; Perez, M.V.; Gomez-Reino, C. (2006). Crossover interconnects in gradient-index planar optics. *Optics Communications*. *266* (2): 490–494.

[16] Bahaa E. A. Saleh, Malvin Carl Teich. (1991). *Fundamental of photonics, Chap: Nonlinear optics*. Series Editor(s): J. W. Goodman, Copyright © 1991 John Wiley & Sons, Inc, 752.

[17] A.D. Polyanin, A. V. Manzhirov. (2016). *Handbook of Methematics for Engineers and Scientics*. Chapman & Hall/CRC, Taylor & Francis Group.

[18] Rainer Schuhmann. (2019). Description of aspheric lens. *Adv. Opt. Chechn. 8*, 267-278.

[19] Jose Sasian. (2019). *Introduction to lens design*. Cambridge University Press.

[20] Pruss, Christof; et al. (2008), Testing aspheres, *Optics & Photonics News*. 19 (4), 19-24.

[21] Forbes, Greg. (2007). Shape specification for axially symmetric optical surfaces. *Opt. Express. 15* (8): 5218–5226.

[22] Bahaa E. A. Saleh, Malvin Carl Teich. (1991). *Fundamental of photonics, Chap: Optic fibers.* Series Editor(s), J. W. Goodman, Copyright © 1991 John Wiley & Sons, Inc, 288.

[23] Bahaa E. A. Saleh, Malvin Carl Teich. (1991). *Fundamental of photonics, Chap: Ray Optics.* Series Editor(s), J. W. Goodman, Copyright ©. (1991) John Wiley & Sons, Inc, 27.

[24] M. Ruphuy, and O. M. Ramahi. (2016). Spherical aberration in electrically thin flat lenses. *J. of OSA A Vol. 33*, No. 8, 1531.

[25] Scott Sparrold, Anna Lansing. (2011). Spherical aberration compensation plates. *Opt. Comp. Photonic international-2011*, *Originally published in German in Photonic 2/2011*, 23-35.

[26] David C. Compertore, Filipp V. Ignatovich, Matthew E. Herbrand, Michael A. Marcus. (2013). Development of a Calibration Standard for Spherical Aberration. *Proceedings of SPIE - The International Society for Optical Engineering, September 2013,*DOI: 10.1117/12.2032535.

[27] P. D. Quy, N.A. Tuan, M. N. Cong, P. M. Cuong, Kh. H. Trong, and N. M. Duong. (2022). Calculation and design of thermal object lens using aspherical lens for handheld observation devices. *J. of Military Scien. and Techn*. No. 76, 127-136.